Experimental Construction of Optical Multi-qubit Cluster States From Bell States

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Cluster states serve as the central physical resource for the measurement-based quantum computation. We here present a simple experimental demonstration of the scalable cluster-state-construction scheme proposed by Browne and Rudolph. In our experiment, three-photon cluster states are created from two Bell states using linear optical devices. By observing a violation of three-particle Mermin inequality of $|\langle A\rangle| = 3.10 \pm 0.03$, we also for the first time report a genuine three-photon entanglement. In addition, the entanglement properties of the cluster states are examined under σ_z and σ_x measurements on a qubit.

There has been considerable interest in optical approaches to quantum computation due to photon's intrinsic robustness against decoherence and the relatively ease of manipulation with high precision. Remarkably, by exploiting the nonlinearity induced by measurement, Knill, Laflamme, and Milburn showed that efficient quantum computation is possible with linear optics [1]. A number of simplifications and modifications [2, 3] of this scheme, as well as experimental demonstrations of the most elementary components [4, 5, 6, 7] have been reported.

Surprisingly, Raussendorf and Briegel proposed a conceptually new quantum computation model [8]. They have shown that universal quantum computation can be done by one-qubit measurements on a specific entangled state, the cluster state [9]. With the cluster states prepared, information is then written onto, processed, and read out from the cluster by one-particle measurement only. After a sequence of one-qubit measurement which forms the computational program, the entanglement in a cluster state is destroyed. Therefore this scheme was called as "one-way quantum computer" or "measurement-based quantum computer". Underlying this novel computation model is the cluster states, serving as the entire physical resources. Many efforts have been devoted to constructing the cluster states. Proposals and experiments using neutral atoms trapped in the periodic potential of an optical lattice with controlled collisions between neighboring atoms have been reported [10]. On the optical approach, Nielsen [3] showed that optical cluster states can be efficiently created using nondeterministic gates from the KLM scheme. Recently, a much more simple and powerful linear optical quantum computation scheme was proposed by Browne and Rudolph [11]. They showed how cluster states may be efficiently generated from pairs of maximally entangled photons in a scalable way using some technique called qubits "fusion". It is significantly less demanding not only in resource requirement, but also in complexity of experimental implementation.

In this letter, we report the first experimental demonstration of constructing linear multi-qubit cluster states from pairs of Bell states. As the most fundamental step in Browne and Rudolph's scheme, we produced a three-photon cluster states by a Type-I "fusion" of two pairs of maximally polarized entangled photons. We then provide sufficient experimental evidence confirming that the cluster states we obtained, unitarily equivalent to the three-photon Greenberger-Horne-Zeilinger (GHZ) states [12], are genuine three-particle entanglement, thus excluding any possibility of hybrid models [13]. We also examined the entanglement properties of the remaining two photons under a measurement on a qubit in different basis $\{\sigma_z, \sigma_x\}$.

Let us first review Browne and Rudolph's efficient linear optical quantum computation scheme. The primary resource used is two photon Bell states which are relatively easier to obtain, probabilistically from single photons for example. Given a supply of Bell states, arbitrarily long linear cluster states can be generated efficiently using an operation called Type-I qubit "fusion" (see Fig.1(a)). This operation, the same as parity check [14, 15], is implemented by mixing two photons in a polarizing beam splitter (PBS) and accepting the output in mode $3'$ only for those cases in which polarization-sensitive detector D1 receives one and only one photon.

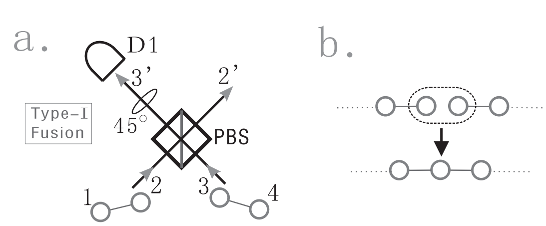

FIG. 1: (a). non-deterministic qubit "fusion" operation. D1 stand for a polarization discriminating photon detector. Two photons of different spatial modes are mixed in a PBS, the output in mode $3'$ is accepted only when D1 receives exactly one photon. (b). A success Type-I "fusion" combines two linear clusters of length n and m into a new one of length (n+m-1).

Here is the most simple and fundamental case: from two pairs of Bell states to a three-qubit cluster state. Encoded in polarization, a Bell state, also equivalent to a two-qubit cluster states under a local unitary transformation can be written as:

$$|\phi\rangle_{ij} = |H\rangle_i |H\rangle_j + |V\rangle_i |V\rangle_j =_{l.u.} |H\rangle_i |+\rangle_j + |V\rangle_i |-\rangle_j$$

Here H and V denote horizontal and vertical polarizations, $|\pm\rangle = \frac{1}{\sqrt{2}}|H\rangle \pm |V\rangle$; i and j index the photon's spatial modes. Given two pairs of Bell states, $|\phi\rangle_{12}$ and $|\phi\rangle_{34}$, we then superpose photon 2 and 3 in a PBS. Since the PBS transmits horizontal and reflects vertical polarization, detecting one and only one photon in D1 makes sure that both photon 2 and photon 3 are horizontally polarized or vertically polarized. By a further measurement performing on output $2'$ in the $+/-$ basis, photon 1, 3, 4 will be in a three-photon cluster state:

$$|\phi^{\pm}\rangle_{134} = |+\rangle_1 |H\rangle_3 |+\rangle_4 \pm |-\rangle_1 |V\rangle_3 |-\rangle_4$$

depending on the measurement result of detector D1. It is equivalent to a three-qubit GHZ state [12] under local unitary transformation.

As has been discussed in detail by Browne and Rudolph in Ref. [11], the creation of three-photon cluster states is the most fundamental step towards the goal of constructing a square lattice cluster state that would allow a simulation of arbitrary quantum network directly by single-qubit measurement alone [8]. With a success probability of 50%, Type-I "fusion" combined two linear cluster state of lengths n and m into a new one of length (n+m-1). Any linear cluster states with desired length can be efficiently created with this method given necessary resource of Bell states. Further, we can also generate arbitrary two-dimensional cluster from those obtained linear cluster states by some similar methods [16].

We note that, compared to the previous schemes, it not only reduces the resources required but also moves away the difficulty of interferometric phase stability. And there are also further advantages reported in Ref. [11]. Obviously, given perfect photon pairs and number-discriminating photon detectors, the scheme described above can be realized optimally without postselection. However, we note that, although these techniques are not available yet, it is still sufficient to perform an experimental demonstration based on postseletion.

A schematic drawing of our experimental setup is shown in Fig.2. An ultraviolet pulsed laser from a mode-locked Ti:sapphire laser (center wavelength 394nm, pulse duration 200fs, repetition rate 76MHz) passed through β-barium borate (BBO) crystal twice to generate two maximally entangled photon pairs in mode 1-2 and mode 3-4. After proper birefringence compensation and local unitary transformation with half wave plate (HWP) and nonlinear crystals, two pairs of two-qubit cluster states are produced as the primary source.

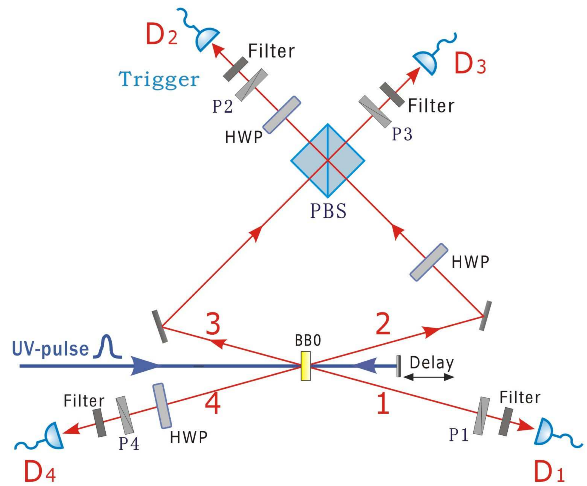

FIG. 2: Experimental setup of generating three-photon cluster state from two pairs of maximally entangled photons produced by Type-II spontaneous parametric down-conversion. The half wave plate (HWP) used in path 2 and 4 are used to locally transform the photon from H/V basis to $+/-$ basis and the four polarizers P1, P2, P3, P4 are used for necessary polarization analysis. In the experiment, we managed to obtain an average twofold coincidence of $2.2 \times 10^4 s^{-1}$.

We then superpose the photon 2 and photon 3 at the PBS. Their path lengths are adjusted such that they arrive simultaneously. To achieve good spatial and temporal overlap, the outputs are spectrally filtered ($\triangle\lambda = 2.8nm$) and monitored by fiber-coupled single-photon detectors. The filtering process stretches the coherence time to about $740fs$, substantially larger than the pump pulse duration [17]. There processes effectively erase any possibility to distinguish the two photons and subsequently lead to interference.

To experimentally verify the three-photon cluster state, we first show that, upon a trigger of D2, the three-fold coincidence only includes $+H+$ and $-V-$ components, but no others. This is done by comparing the counts of all 8 possible polarization combination $+H+, \cdots, -V-$. The experimental results in the $(+/-, H/V, +/-)$ basis (see Fig.3(a)) show that the signal-to-noise ratio defined as the ratio of any of the desired threefold events ($+H+$ and $-V-$) to any of the six other undesired ones is about $29:1$ on average. Second, we further perform a polarization measurement in the "diagonal" basis $(H/V, +/-, H/V)$ to demonstrate that the two terms $+H+$ and $-V-$ are indeed in a coherent superposition. Transform $|\phi^{+}\rangle_{123}$ in the "diagonal" basis $(H/V, +/-, H/V)$, we note that only components $(H+H, H-V, V+V, V-H)$ occur, other combinations $(H-H, H-V, V+H, V-V)$ do not occur. As a test for coherence, we compare the H+H and H-H count rates as a function of the pump delay mirror position. It shows in Fig.3(b) that, at zero delay, the unwanted component is

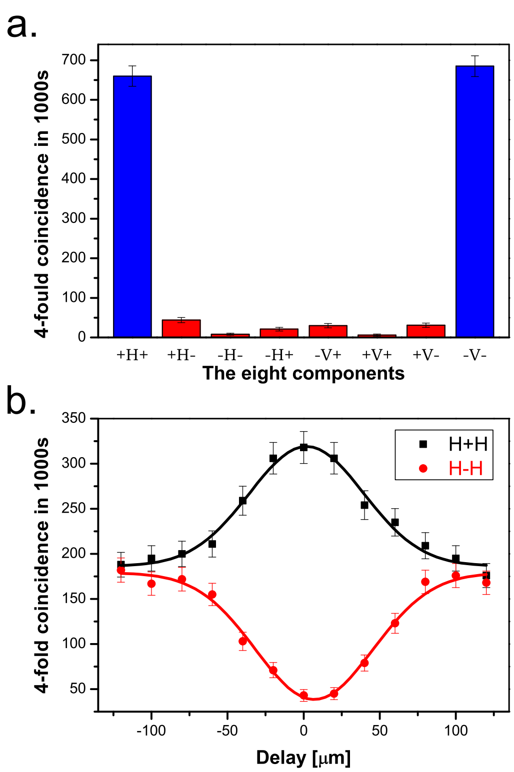

FIG. 3: (a) Experimental data under 8 different polarizing settings. Two desired terms $+H+$ and $-V-$ are prominent while other six are strongly depressed to be about 3% of any desired ones on average. (b) Experimental data in the "diagonal" basis showing the two components are in a coherent superposition. Maximum interference occurs at zero delay between the two incoming photons.

suppressed with a visibility of 0.78 ± 0.03, which is sufficient to violate the Bell-type inequality imposed by local realism [18].

However, as in the previous experiments of Bouwmeester *et al.* [19] and Pan *et al.* [20], the data presented above are still not sufficient to confirm the genuine entanglement of all three particles [13]. This has been shown by M. Seevinck and J. Uffink that it can be explained by a hybrid model in which only less than three particles is entangled. Aim to exclude such a hybrid model and produce the three-photon GHZ state in the form $|HHH\rangle + |VVV\rangle$, we first did a local transformation of the cluster state and performed four series of measurements in the $\sigma_x\sigma_x\sigma_x, \sigma_x\sigma_y\sigma_y, \sigma_y\sigma_y\sigma_x$ and $\sigma_y\sigma_x\sigma_y$ direction. We then test the three-particle Bell inequality of the form derived by Mermin [21] and the result shows:

$$|\langle A\rangle| = 3.10 \pm 0.03$$

where

$$A = \sigma_x\sigma_y\sigma_y + \sigma_y\sigma_x\sigma_y + \sigma_y\sigma_y\sigma_x - \sigma_x\sigma_x\sigma_x$$

It clearly shows a violation of the inequality: $|\langle A\rangle| \leqslant 2$ imposed by local realism by 34 standard deviations. As has been discussed in Ref. [13], confirmation of genuine three-particle entanglement requires a violation of inequality: $|\langle A\rangle| \leqslant 2\sqrt{2}$. The experimental result also well exceeds the bound to confirm genuine three-photon entanglement, with a violation of this inequality by 11 standard deviations, hence leads to verification of genuine three-photon entanglement. Our three-photon entanglement source thus distinguishes itself from all previous ones by its high purity, which would make it possible to perform a lot of quantum information processing tasks, such as quantum secreting sharing and the third-man cryptography [22, 23].

A quite interesting entanglement property of a linear

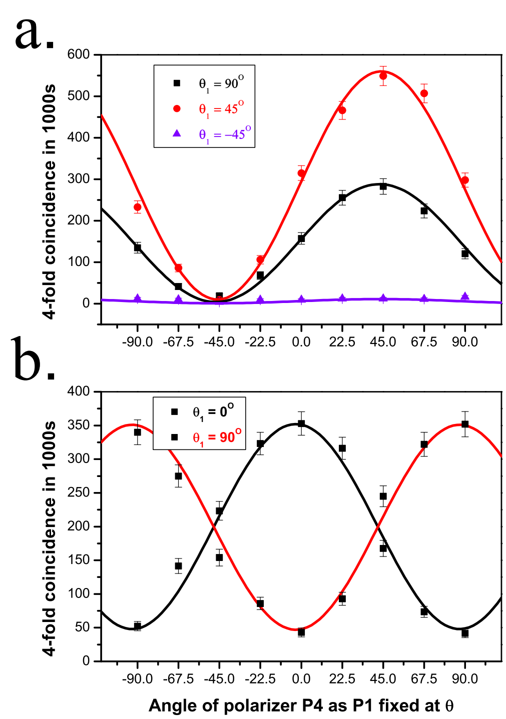

FIG. 4: Experimental results showing polarization correlation between photon 1 and 4, under a σ_z and σ_x measurement of photon 3. (a). Data obtained under a σ_z measurement. The coincident counts when P1 was set at $-45°$ was so strongly suppressed that they can hardly concerned; while counts when P1 was set at $45°$ are the most prominent, twice as when P1 was set at $90°$. The experimental results clearly agree that what we obtained is $|+\rangle_1 \otimes |+\rangle_4$. (b). Data obtained under a σ_x measurement. The two sinusoidal curves with a visibility of 0.79 ± 0.03 demonstrate that photon 1 and 4 are in an entangled state as $|+\rangle_1|+\rangle_4 + |-\rangle_1|-\rangle_4$.

cluster state is that, measurements in σ_z and σ_x basis on a qubit of a cluster state have totally different effects on the remaining qubits. This has been shown in Ref. [11] that, a σ_z eigenbasis measurement removes the qubit from the cluster sand breaks all bond between that qubit and the rest of the cluster; while a σ_x measurement on a linear cluster removes the measured qubit and it combines the adjacent qubits into a redundantly encoded qubit. It is quite critical for us to understand the cluster-state-construction scheme [11] and the cluster model of quantum computation [8]. We then examined the entanglement properties of the two remaining photons under a σ_z measurement and a σ_x measurement on the "mid" qubit upon trigger of detecting a $|H\rangle$ photon and a $|+\rangle$ photon by D3 respectively. We analyzed the polarization correlations between photon 1 and 4 by keeping polarizer 1 fixed and varying the angle of polarizer 4. The experimental results are shown in Fig.4(a) and Fig.4(b) corresponding to measurement in the σ_z and σ_x basis respectively. As Fig.4 shows, the experimental data are in good agreement with theoretical prediction.

In summary, we have demonstrated the process of constructing linear three-photon cluster state from two Bell states. In principle these method can be extended to any desired number of particles given enough Bell states, which holds the promise of constructing an optical one-way quantum computer efficiently. Our experiment can also be considered as a demonstration of producing a genuine three-photon GHZ state [24] in an event-ready way, which in principle does not need postselection given perfect photon pairs and perfect detectors. The genuineness of the three-photon entangled state was confirmed by violating the inequality: $|\langle A\rangle| \leqslant 2\sqrt{2}$ by 11 standard deviations. After verification of the obtained three-photon cluster state, we also demonstrate that a σ_z measurement on a qubit of the obtained three-photon cluster state breaks the bond between the rest photons; while a σ_x measurement does not, but instead combines them into a redundantly encoded qubit. However, in this experiment, only partial features of cluster states and the cluster-state-construction scheme were demonstrated. Possible future work could include production and characterization of a four-photon cluster state, which is local unitary inequivalent to four particle GHZ state and has a higher entanglement persistency [9] and use the obtained cluster state to implement some interesting quantum computation tasks. By exploiting photon's intrinsic flying nature, we could also envision that this experimental technique maybe applicable in distributed quantum computation and "quantum internet" [25]. When combined with recent advance in neutral atoms trapped in an optical lattice [10] and atom-photon entanglement [25], we could also dream of a photon-assisted atomic one-way quantum computer that can efficiently implement distributed quantum information processing. We expect this work would stimulate further work towards feasible quantum computation. In any event, the experimental results present here may provide a first step towards that goal.

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